JMEPEG (2011) 20:169–176
DOI: 10.1007/s11665-010-9667-4

1059-9495/$19.00

A New Three-Parameter Model for Predicting Fatigue Crack Initiation Life

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(Submitted May 22, 2008; in revised form February 25, 2010)

Based on the idea that the fatigue damage is caused by the cyclic damage strain, a concept of the critical damage quantity is introduced and a new three-parameter model is developed. The model contains three material performance parameters, i.e., the fatigue ductility coefficient, the fatigue ductility exponent, and the theoretical strain endurance limit. The fatigue ductility coefficient reflects the existence of the critical damage quantity. The fatigue ductility exponent shows the damage resistance ability of the material. And the theoretical strain endurance limit represents the existence of the critical cyclic strain. By using the proposed model, the fatigue crack initiation life of metallic materials can be predicted.

Keywords the critical damage quantity, the cyclic damage strain, the fatigue crack initiation life, the theoretical strain endurance limit, the three-parameter model

1. Introduction

In order to exactly predict the fatigue crack initiation life of metallic materials, a proper expression of fatigue crack initiation life is necessary. In most cases, the Manson-Coffin's equation (Ref 1-3) is used to predict the fatigue crack initiation life. However, when cycle number $N_i > 10^6$, the Manson-Coffin's equation usually underestimates the fatigue crack initiation life because it does not reflect the existence of the theoretical strain endurance limit (Ref 4, 5). To solve this problem, a new idea has been introduced by Zheng and his co-workers (Ref 5-9), that is, the cyclic strain can be considered as two parts, i.e., the cyclic damage strain and the cyclic non-damage strain. The main factor causing the fatigue damage is not the cyclic plastic strain but the cyclic damage strain. Based on this idea, Zheng et al. established a formula to relate the fatigue crack initiation life with the cyclic damage strain (Ref 5-9). This formula contains two material performance parameters, i.e., the fatigue ductility coefficient and the theoretical strain endurance limit. For sake of describing clearly, the formula is called as two-parameter model for the time being.

By comparing the two-parameter model with the Coffin's equation (Ref 10, 11), it can be found that the cyclic damage strain in the former replaces the cyclic plastic strain in the latter, while the fatigue ductility exponents and the fatigue ductility coefficients in the two formulae are the same. It will be seen in the following study that neither the meaning of the cyclic damage strain nor the value of it is the same as the cyclic plastic strain, the fatigue ductility exponents and the fatigue ductility coefficients in the two formulas should not be the same values. Therefore, in the present article, the idea of the cyclic damage strain which produces the fatigue damage is accepted, but the fatigue ductility exponent and the fatigue ductility coefficient are not taken as constants. In addition, a concept of the critical damage quantity is introduced and a new model is developed. This new model contains three parameters, and therefore, is called as three-parameter model. The main characteristics of the three-parameter model are that it reflects the existence of the critical damage quantity and shows the damage resistance ability of the material. By using the three-parameter model, the fatigue crack initiation life of metallic materials can be predicted.

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2. Traditional Models for Predicting Fatigue Crack Initiation Life

It is well known that the local cyclic plastic strain is the basic factor to induce the fatigue damage (Ref 4). Based on this idea, Coffin et al. established the correlation between the

Nomenclature	
N_i	fatigue crack initiation life
$\Delta\varepsilon_p$	cyclic plastic strain range
$\Delta\varepsilon_D$	cyclic damage strain range
c	fatigue ductility exponent
σ_f'	fatigue strength coefficient
ε_f	fracture ductility
σ_{-1}	stress endurance limit
$\Delta\varepsilon$	cyclic strain range
$\Delta\varepsilon_e$	cyclic elastic strain range
$\Delta\varepsilon_c$	theoretical strain endurance limit
ε_f'	fatigue ductility coefficient
b	fatigue strength exponent
σ_b	ultimate tensile strength
E	Young's modulus

fatigue crack initiation life and the cyclic plastic strain range, and the expression of the correlation is (Ref 10, 11):

$$\Delta\varepsilon_p N_i^{0.5} = C \quad \text{(Eq 1)}$$

Equation 1 is the so-called Coffin's equation. Where N_i is the fatigue crack initiation life, $\Delta\varepsilon_p$ is the cyclic plastic strain range, C is a material constant. Coffin's equation can only be used in low-cycle fatigue (Ref 10-12). For high-cycle fatigue, the Manson-Coffin's equation should be used (Ref 1):

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_i)^b + \varepsilon_f'(2N_i)^c \quad \text{(Eq 2)}$$

where $\Delta\varepsilon$ is the cyclic strain range, E is the Young's modulus, σ_f' is the fatigue strength coefficient, b is the fatigue strength exponent, ε_f' is the fatigue ductility coefficient, and c is the fatigue ductility exponent. In the absence of the four fatigue parameters, the Manson-Coffin's equation can be approximately replaced by (Ref 2, 3):

$$\Delta\varepsilon = \frac{3.5\sigma_b}{E}N_i^{-0.12} + \varepsilon_f^{0.6}N_i^{-0.6} \quad \text{(Eq 3)}$$

where σ_b is the ultimate tensile strength, ε_f is the fracture ductility. As can be seen from Eq 2 and 3, only if $\Delta\varepsilon \to 0$ then $N_i \to \infty$. That is to say, Eq 2 and 3 do not reflect the existence of the endurance limit. In fact, as the same as Coffin's equation, Eq 2 and 3 usually underestimate the fatigue crack initiation life when $N_i > 10^6$ (Ref 4, 5).

In order to amend the deviation of the Manson-Coffin's equation in high-cycle range, the following relation was introduced for St 37 steel, HSB77V and HSB55C (Ref 4):

$$N_i^k(\Delta\varepsilon_p + B) = K \quad \text{(Eq 4)}$$

In Eq 4, k, B, K are all material constants determined by experiments. And B represents $\Delta\varepsilon_p$ corresponding to the endurance limit. For St 37 steel, Eq 4 changes into (Ref 4):

$$N_i^{0.585}(\Delta\varepsilon_p - 0.06\times10^{-3}) = 10^3. \quad \text{(Eq 5)}$$

For polycrystalline metals, disordered crystal orientations and inhomogeneous microstructures make the plastic deformations occur in some grains even if the external loading is lower than the elastic limit. However, for Armco-iron (Ref 4), under a determinate cyclic stress, the local plastic strain has not been found until 10^8 cycles. For Cu-Al single crystal alloy (Ref 13), the fatigue cracks cannot be found when cyclic stress is lower than a definite value. And for aluminum (Ref 5), under a definite cyclic stress, the fatigue failure has not happened until 5×10^9 cycles. These facts show that, for a material, a critical cyclic stress or a cyclic strain exists. When loaded cyclic stress or loaded cyclic strain is smaller than the critical value, the fatigue damage will not occur. In this case, the fatigue life of the specimen will be infinite. This critical cyclic strain (e.g., $\Delta\varepsilon_p$ equals to 0.06×10^{-3} for St 37 steel, see Eq 5) is defined as the theoretical strain endurance limit (Ref 5-9). The difference between the cyclic strain and the theoretical strain endurance limit is defined as the cyclic damage strain. It is the cyclic damage strain that is the basic factor to produce the fatigue damage (Ref 5-9).

Based on the idea above, Zheng and coworkers (Ref 5-9) assumed that the fatigue crack initiation life should be related to the cyclic damage strain range. This relation is expressed as (Ref 5-9):

$$\Delta\varepsilon_D = \Delta\varepsilon - \Delta\varepsilon_c = \varepsilon_f' N_i^{-0.5} \quad \text{(Eq 6)}$$

where $\Delta\varepsilon_D$ is the cyclic damage strain range, $\Delta\varepsilon_c$ is the theoretical strain endurance limit, and ε_f' is the fatigue ductility coefficient. Equation 6 contains two parameters, ε_f' and $\Delta\varepsilon_c$, so we call it as two-parameter model.

The test results showed (Ref 10, 11) that Eq 1 holds in the case of N_i = 1/4 and $\Delta\varepsilon_p = 2\varepsilon_f$, which is the case of monotonic tension. Based on this fact and on that $\Delta\varepsilon_p \approx \Delta\varepsilon_D$ in low-cycle fatigue, Zheng and coworkers (Ref 5-9) assumed that:

$$\Delta\varepsilon_D = 2\varepsilon_f \quad \left(\text{for } N_i = \frac{1}{4}\right) \quad \text{(Eq 7)}$$

From Eq 6 and 7, Zheng et al. obtained that:

$$\varepsilon_f' = \varepsilon_f \quad \text{(Eq 8)}$$

As for the theoretical strain endurance limit, $\Delta\varepsilon_c$, the following expression is given (Ref 5-9):

$$\Delta\varepsilon_c = \frac{2\sigma_{-1}}{E} - 10^{-3.5}\varepsilon_f \quad \text{(Eq 9)}$$

where σ_{-1} is the stress endurance limit. Taking Eq 8 and 9 into Eq 6, then:

$$\Delta\varepsilon_D = \Delta\varepsilon - \left(\frac{2\sigma_{-1}}{E} - 10^{-3.5}\varepsilon_f\right) = \varepsilon_f N_i^{-0.5} \quad \text{(Eq 10)}$$

Table 1 displays 18 metallic materials' performance parameters (Ref 14-19). Using Eq 10 and the data in Table 1, $\Delta\varepsilon/2 \sim 2N_i$ curves can be obtained (see Fig. 1-18). Comparing the $\Delta\varepsilon/2 \sim 2N_i$ curves with the strain fatigue test data given by Ref. 14-19, we can find that the predicted results by Eq 10 are close to the test data for some materials such as GH4133 alloy steel, Ti-6Al-4V and Ti-8Al-1Mo-1V titanium alloys. But for the other materials, such as LY12CZ(rod), LY12CZ(plate), and 7075-T6 aluminum alloys, the predicted results are deviate significantly from the test data. The reason for that may mainly lie in the inappropriate values of the fatigue ductility exponent and the fatigue ductility coefficient in Eq 10.

Table 1 The metallic materials' performance parameters

Materials	E, MPa	ε_f, %	σ_{-1}, MPa
GH4133	199200	34.3	420.7
GH36	203000	38.6	333.2
30CrMnSiA	203005	77.3	588.5
30CrMnSiNi2A	200063	74.0	827.7
40CrMnSiMoVA	200455	63.3	937.7
AISI4340	193060	84.0	620.5
LY12CZ (rod)	73160	18.0	78.9
LY12CZ (plate)	71022	30.1	72.4
LC4CS	72572	18.0	85.1
LC9CgS3	72180	28.3	80.3
2014-T6	68950	29.0	75.7
2024-T4	70329	43.0	72.5
7075-T6	71018	41.0	82.0
Ti-6Al-4 V	117215	53.0	416.0
Ti-8Mo-1Mo-1	117215	66.0	309.4
BT20L1	115700	51.0	379.2
BT20T1	121000	56.2	335.3
BT20T2	118300	42.8	327.3

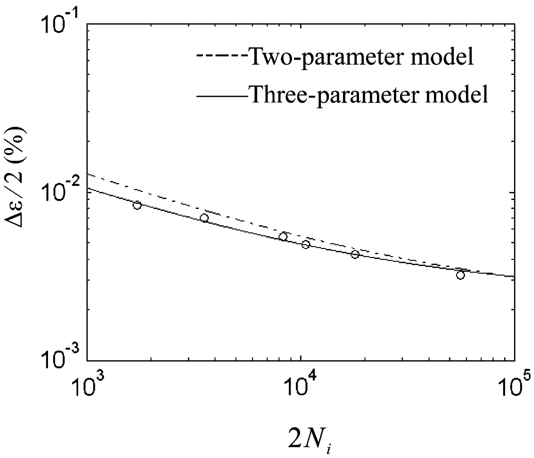

Fig. 1 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of GH4133

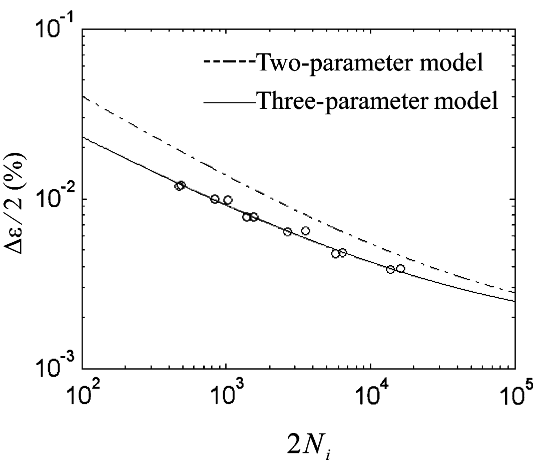

Fig. 2 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of GH36

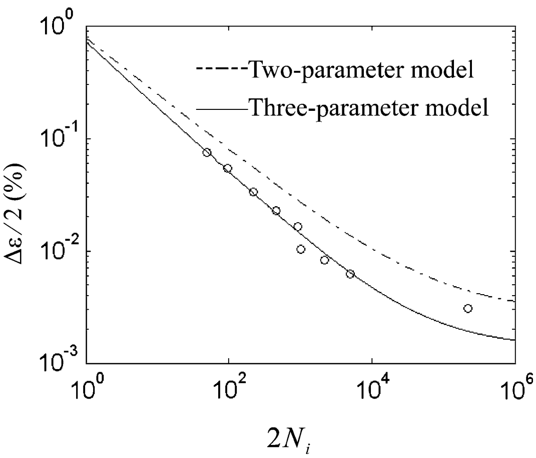

Fig. 3 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of 30CrMnSiA

On the one hand, neither the meaning of the cyclic damage strain nor the value of it is the same as the cyclic plastic strain. The expressions of the cyclic damage strain and the cyclic plastic strain are that:

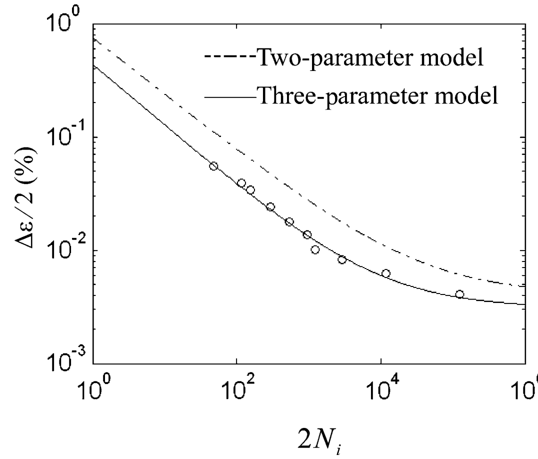

Fig. 4 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of 30CrMnSiNi2A

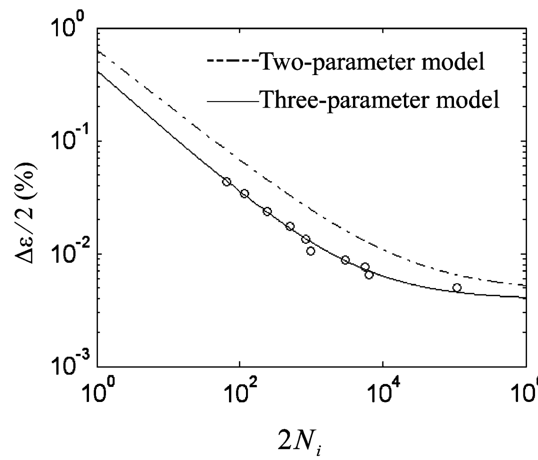

Fig. 5 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of 40CrMnSiMoVA

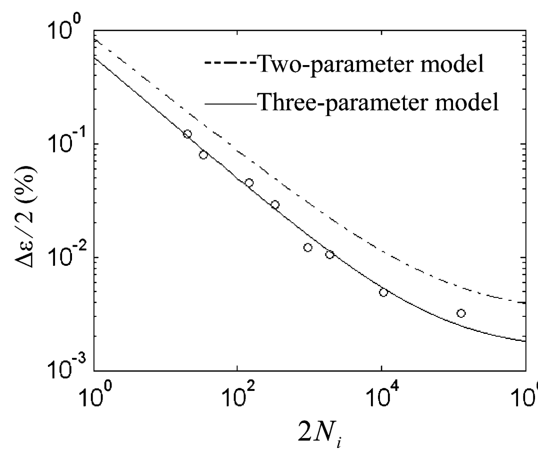

Fig. 6 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of AISI4340

$$\Delta\varepsilon_D = \Delta\varepsilon - \Delta\varepsilon_c \qquad \Delta\varepsilon_p = \Delta\varepsilon - \Delta\varepsilon_e \qquad \text{(Eq 11)}$$

where $\Delta\varepsilon_e$ is the cyclic elastic strain range. As mentioned above, for polycrystalline metals, disordered crystal

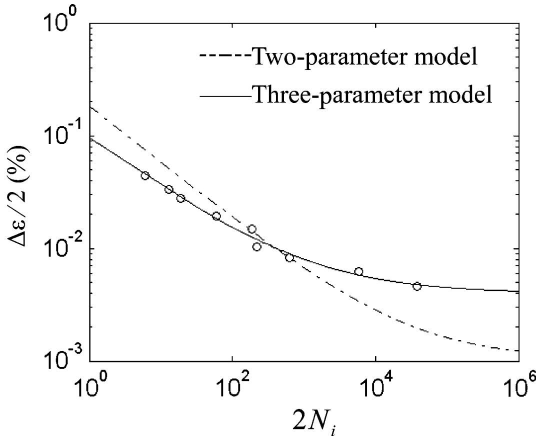

Fig. 7 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of LY12CZ(rod)

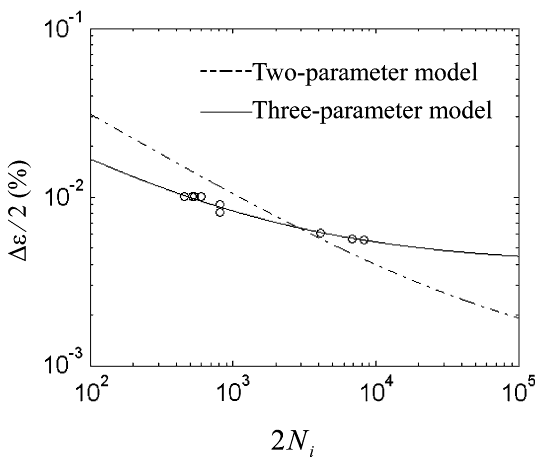

Fig. 8 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of LY12CZ(plate)

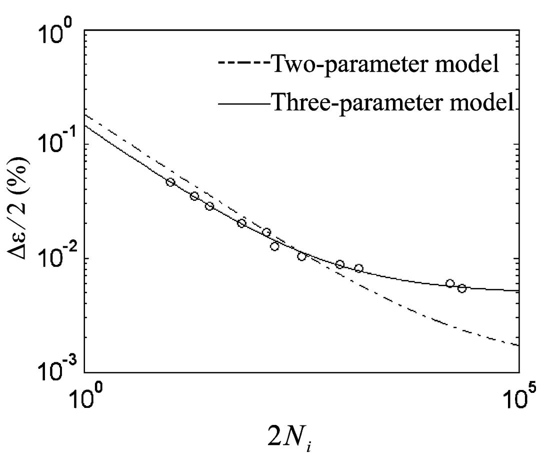

Fig. 9 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of LC4CS

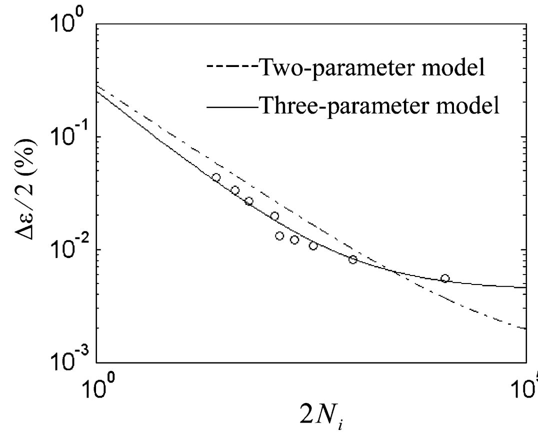

Fig. 10 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of LC9CGS3

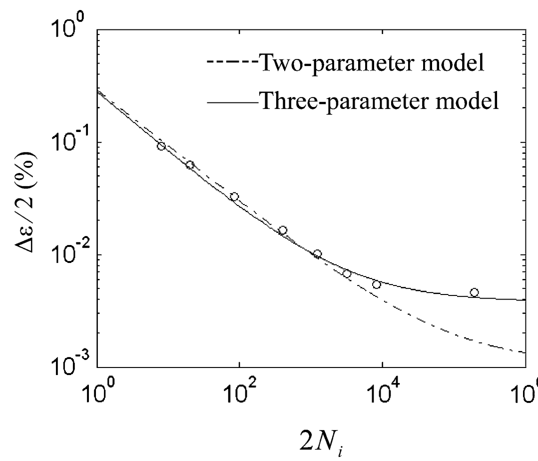

Fig. 11 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of 2014-T6

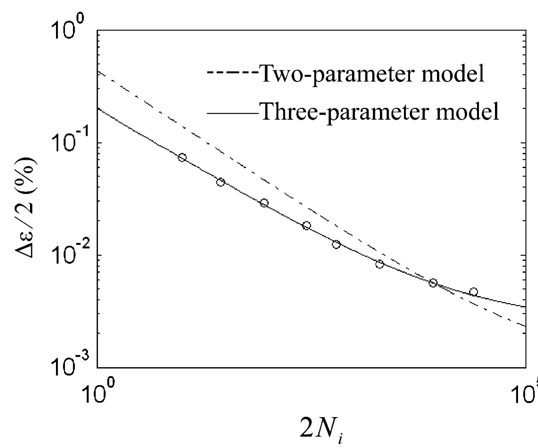

Fig. 12 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of 2024-T4

orientations and inhomogeneous microstructures make the plastic deformations occur in some grains even if the external loading is lower than the elastic limit. While the theoretical strain endurance limit means that there is no fatigue damage when the cyclic strain range is smaller than this limit. Thus, to a definite cyclic strain, the theoretical strain endurance

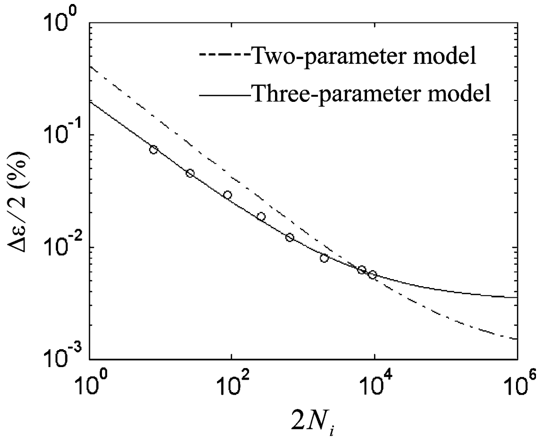

Fig. 13 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of 7075-T6

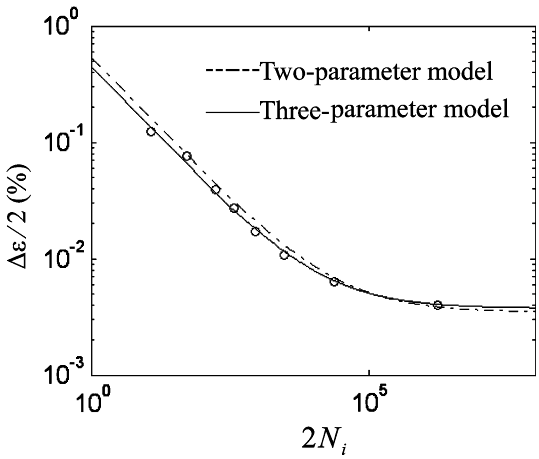

Fig. 14 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of Ti-6Al-4V

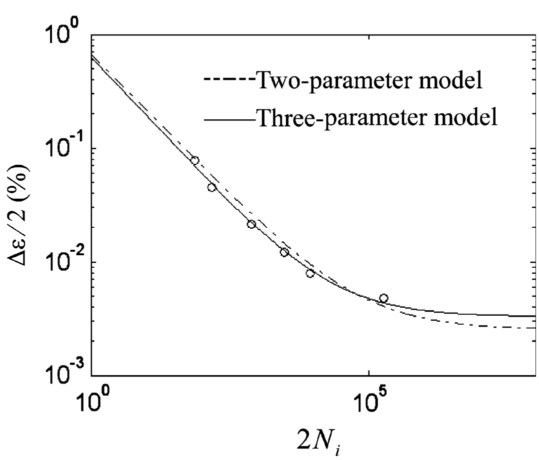

Fig. 15 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of Ti-8Al-1Mo-1V

limit is smaller than the cyclic elastic strain range, and the cyclic damage strain is greater than the cyclic plastic strain. On the other hand, the magnitudes of the fatigue ductility exponent and the fatigue ductility coefficient in the Coffin's

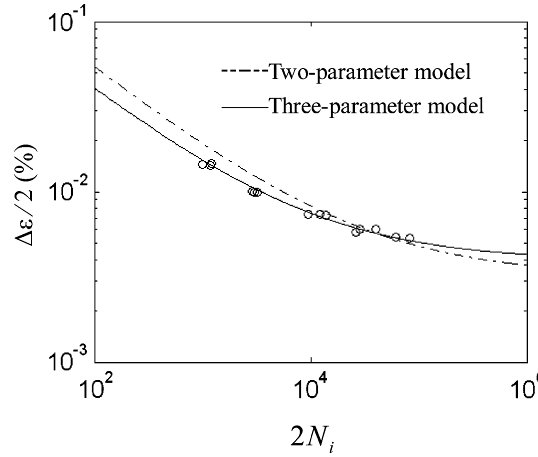

Fig. 16 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of BT20L1

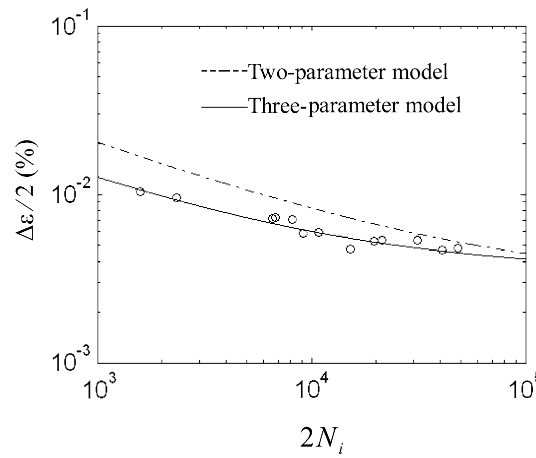

Fig. 17 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of BT20T1

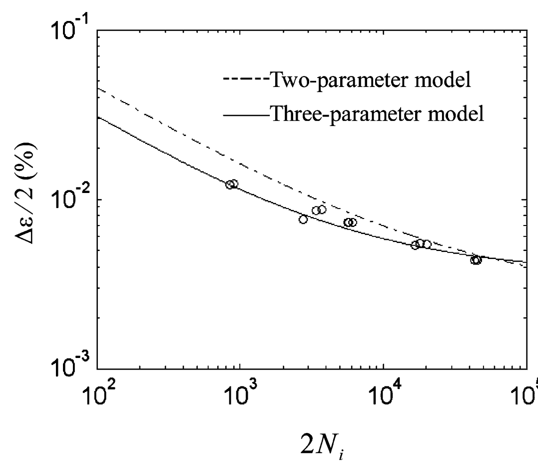

Fig. 18 $\Delta\varepsilon/2 \sim 2N_i$ curves and the test data of BT20T2

equation are all variables (Ref 4, 12), and the variation ranges are:

$$0.25 < c < 1.0 \qquad \frac{1}{2}\varepsilon_f < C < \frac{1}{\sqrt{2}}\varepsilon_f \qquad \text{(Eq 12)}$$

The true value of the two parameters depends on material, environmental conditions and the yielding criterion (Ref 12). Therefore, 0.5, the value of the fatigue ductility exponent and ε_f, the value of the fatigue ductility coefficient in the two-parameter model (see Eq 10), are not proper. The expression of the two-parameter model, Eq 6 and 10, should be corrected.

3. A New Three-Parameter Model for Predicting Fatigue Crack Initiation Life

As in Ref 5-9, it is assumed that the theoretical strain endurance limit always exists and that the cyclic damage strain is the basic factor resulting in the fatigue damage. By contrast, the relationship between the fatigue crack initiation life and the cyclic damage strain range is assumed to be:

$$\frac{\Delta\varepsilon_D}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2} = \varepsilon_f'(2N_i)^{-c} \qquad \text{(Eq 13)}$$

Also, in Eq 13, ε_f' is the fatigue ductility coefficient, c is the fatigue ductility exponent, but they are not equal to those in Eq 2 and 6. There are three parameters in Eq 13, thus, formula 13 is called as three-parameter model. This model is somewhat similar to formula 4.

In order to visually display the meanings of the fatigue ductility coefficient ε_f' and the fatigue ductility exponent c in the three-parameter model, Eq 13 is changed into:

$$\log\frac{\Delta\varepsilon_D}{2} = \log\left(\frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2}\right) = \log\varepsilon_f' - c\log 2N_i \qquad \text{(Eq 14)}$$

Relation 14 shows that, in bi-logarithm coordinate system as shown in Fig. 19, the fatigue crack initiation life changes linearly with the cyclic damage strain range. ε_f' is the intercept of the line on the $\log\left(\frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2}\right)$ axis and c is the slope.

Compared with the Coffin's equation, formula 13 has a term $\Delta\varepsilon_c/2$. If $\Delta\varepsilon/2 \le \Delta\varepsilon_c/2$, then $N_i \to \infty$, so the three-parameter model reflects the existence of the critical cyclic strain, i.e., the existence of the theoretical strain endurance limit.

In the two-parameter model, the fatigue ductility exponent c equals to 0.5, which means that the slope of the line shown as Fig. 19 is a constant for all materials, the differences among the lines lie in the intercepts on $\log\left(\frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2}\right)$ axis. But, in the three-parameter model, the fatigue ductility exponent and the fatigue ductility coefficient are taken as parameters to be determined, which means that different lines may have different slopes and different intercepts on the $\log\left(\frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2}\right)$ axis.

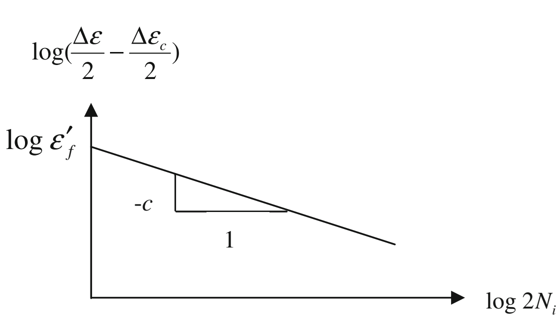

Fig. 19 The fatigue crack initiation life changes with the cyclic damage strain

In order to further explain the meaning of the three-parameter model, Eq 13 is rewritten as:

$$\frac{\Delta\varepsilon_D}{2}(2N_i)^c = \left(\frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2}\right)(2N_i)^c = \varepsilon_f' \qquad \text{(Eq 15)}$$

If a concept of the critical damage quantity is introduced and it is denoted as ε_f', then Eq 15 clearly exhibits the meaning of formula 13, that is, the accumulative damage quantity to the material reaches the critical damage quantity ε_f' after N_i cycles under the cyclic damage strain amplitude $\Delta\varepsilon_D/2$ or under the cyclic total strain amplitude $\Delta\varepsilon/2$. This meaning is identical to the basic assumption, i.e., the fatigue damage is controlled by the cyclic damage strain. To a material, no matter how the cyclic damage strain amplitude or the cyclic total strain amplitude affects the cycle number, the critical damage quantity ε_f' leading to the fatigue crack initiation is always a constant. When the accumulative damage reaches the critical damage quantity, the fatigue crack will be initiated. This is the condition to determine whether the fatigue cracks occur or not.

With the aid of the critical damage quantity ε_f', the meaning of the fatigue ductility exponent c can be obtained. For this purpose, Eq 15 is rewritten as:

$$(2N_i)^c = \frac{\varepsilon_f'}{\frac{\Delta\varepsilon_D}{2}} = \frac{\varepsilon_f'}{\frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2}} \qquad \text{(Eq 16)}$$

Equation 16 shows that, for a given cyclic damage strain amplitude $\Delta\varepsilon_D/2$ or a cyclic total strain amplitude $\Delta\varepsilon/2$, the ratio on the right of Eq 16 is a constant. Therefore, on the left of Eq 16, if c is great then cycle number N_i will be small. That is to say, one cycle will produce greater damage. On the contrary, if c is small then cycle number N_i will be great, one cycle produces smaller damage. Therefore, the fatigue ductility exponent shows the damage resistance ability of the material.

4. Fatigue Parameters in the Three-Parameter Model for Some Metals

In Ref 14-19, strain fatigue test data of 18 alloys have been given. If these test data are used to fit the parameters of the three-parameter model, then Fig. 1-18 and Table 2 can be obtained. Figures 1-18 show that the fitted curves can represent the fatigue behavior of 18 metallic materials. In other words, the three-parameter model can be used to describe the relationship between the cyclic damage strain and the fatigue crack initiation life.

Table 2 shows that the fatigue ductility exponent and the fatigue ductility coefficient in the three-parameter model change with the material. And for the 18 materials, the variation ranges are:

$$0.4501 < c < 0.601 \qquad 0.38\varepsilon_f < \varepsilon_f' < 0.95\varepsilon_f \qquad \text{(Eq 17)}$$

Equation 17 corresponds to Eq 12. The difference between Eq 17 and 12 is produced by the difference between $\Delta\varepsilon_D$ in Eq 13 and $\Delta\varepsilon_p$ in formula 1.

To further compare the fatigue ductility coefficients corresponding to the two models, the different forms between Eq 6 and 13 have to be considered, so ε_{f2}' is used to represent ε_f' in the two-parameter model and ε_{f3}' is used to represent the

Table 2 Fitted fatigue parameters for the three-parameter model

Materials	c	ε'_f, %	$\Delta\varepsilon_c/2$ ($\times 10^{-3}$)	$\varepsilon'_f/\varepsilon_f$
GH4133	0.5025	26.59	2.324	0.78
GH36	0.4501	17.18	1.523	0.45
30CrMnSiA	0.5808	71.36	1.368	0.92
30CrMnSiNi2A	0.5424	43.16	3.084	0.58
40CrMnSiMoVA	0.5568	41.2	3.9	0.65
AISI4340	0.5392	57.55	1.476	0.69
LY12CZ (rod)	0.4526	9.17	3.991	0.51
LY12CZ (plate)	0.4743	11.45	3.968	0.38
LC4CS	0.536	14.08	4.879	0.78
LC9CgS3	0.601	24.55	4.35	0.87
2014-T6	0.5394	27.53	3.765	0.95
2024-T4	0.4553	19.75	2.398	0.46
7075-T6	0.4765	19.52	3.244	0.48
Ti-6Al-4 V	0.5031	43.74	3.746	0.83
Ti-8Mo-1Mo-1	0.525	61.88	3.29	0.94
BT20L1	0.5050	37.57	3.941	0.74
BT20T1	0.5388	38.70	3.338	0.69
BT20T2	0.5423	33.20	3.617	0.78

Table 3 Comparison of the fatigue ductility coefficients of the two models

Materials	c	ε'_{f2}, %	ε'_{f3}, %
GH4133	0.5025	26.6	26.6
GH36	0.4501	…	17.18
BT20L1	0.5050	…	37.57
BT20T1	0.5388	…	38.70
BT20T2	0.5423	…	33.20
30CrMnSiA	0.5808	62.0	95.42
30CrMnSiNi2A	0.5424	49.7	59.27
40CrMnSiMoVA	0.5568	41.6	56.02
LY12CZ (rod)	0.4526	15.7	13.40
LY12CZ (plate)	0.4743	19.1	16.48
LC4CS	0.5360	17.7	19.42
LC9CgS3	0.601	24.0	32.37
AISI4340	0.5392	66.8	79.21
2014-T6	0.5394	26.0	37.88
2024-T4	0.4553	39.2	28.81
7075-T6	0.4765	30.7	28.06
Ti-6Al-4 V	0.5031	57.0	61.72
Ti-8Mo-1Mo-1	0.525	75.2	86.01

corresponding one in the three-parameter model. Then the following relations can be obtained:

$$\varepsilon'_{f2} = \varepsilon'_f \tag{Eq 18}$$

$$\varepsilon'_{f3} = 2^{1-c}\varepsilon'_f \tag{Eq 19}$$

ε'_{f2} and ε'_{f3} from relations 18 to 19 are listed in Table 3. This table shows that:

(1) For the same material, ε'_{f2} is not equal to ε'_{f3}. And the difference between them increases with the deviation of the fatigue ductility exponent from 0.5.
(2) 0.5 is a critical quantity for c. If $c > 0.5$ then $\varepsilon'_{f2} < \varepsilon'_{f3}$, but if $c < 0.5$ then $\varepsilon'_{f2} > \varepsilon'_{f3}$.

5. Conclusions

(1) A new three-parameter model for predicting fatigue crack initiation life is:

$$\frac{\Delta\varepsilon_D}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_c}{2} = \varepsilon'_f(2N_i)^{-c}.$$

The meaning of this model is that the accumulative damage quantity to the material reaches the critical damage quantity ε'_f after N_i cycles under the cyclic damage strain amplitude $\Delta\varepsilon_D/2$ or under the cyclic total strain amplitude $\Delta\varepsilon/2$.

(2) In the three-parameter model, the meaning of the fatigue ductility coefficient ε'_f is the critical damage quantity. The condition to produce the fatigue crack depends on this critical damage quantity. When the accumulative damage reaches the critical damage quantity, the fatigue crack will be initiated.
(3) The fatigue ductility exponent c shows the damage resistance ability of the material. The greater the fatigue ductility exponent becomes, the smaller the damage resistance ability is. But the smaller the fatigue ductility exponent is, the greater the damage resistance ability becomes.
(4) The theoretical strain endurance limit $\Delta\varepsilon_c$ means the existence of the critical cyclic strain. When the loaded cyclic strain is smaller than this critical value, the cyclic plastic deformations cannot occur in all grains. In this case, the fatigue damage is not produced, the fatigue crack is not initiated, and the fatigue life of the specimen will be infinite.

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